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1. Introduction. Let V be a Banach space densely and continously imbedded in a real Hilbert space H. Our purpose in this paper is to consider the existence of solutions of the initial value problem

$$
(1.1)\qquad \begin{aligned} &\frac{du}{dt} + Au + G(u) = f, \qquad 0 < t < T, \\ &u(0) = u_0 , \end{aligned}
$$

where A is a monotone operator from V into V', $G: V \to H$ is a continuous mapping and $f: (0,T) \to V'$ is a measurable function.

Problems of this kind has been studied by many authors. The case A is linear was studied by Browder(5) and Pazy(14). The nonlinear case was studied by Attouch & Damlamian(1), Crandall & Nohel(7), Hirano(10), and Vrabie(15, 16). In (15) and (16), Vrabie studied the problem (1.1) under the assumption that A generates a compact semigroup on H, and satisfies

$$
(1.2)\qquad (Ax - Ay, x - y) + c|x - y|^2 \geq \omega\|x - y\|^p \quad \text{for } x, y \in V,
$$

where $c, \omega > 0$, $p \geq 2$ and $\|\cdot\|$, $|\cdot|$ denotes the norms of V and H, respectively.

In this paper, we consider the case G is a compactly continuous mapping from V into V'. Our argument is based on the existence results for pseudo-monotone mappings (cf. (4, 6)).

2. Statement of main results. Let p, q and T be constants such that $T > 0$, $p \geq 2$ and $1/p + 1/q = 1$. V will denote a reflexive Banach space densely and continuously imbedded in a real Hilbert space H. Identifying H with its dual, we have that $V \subset H \subset V'$, where V' is the dual space of V. the norms of V, H and V' are denoted by $\|\cdot\|$, $|\cdot|$ and $\|\cdot\|_*$, respectively. Let (x, y) denote the pairing of an element $x \in V$ and an element $y \in V'$. If $x, y \in H$, then (x, y) is the ordinary inner product of H. Let A be a mapping from V into V'. Then A is called monotone if $(Ax - Ay, x - y) \geq 0$ for x, $y \in V$. the mapping A is said to be hemicontinuous if for each x, $y \in V$, $A(u+tv)$ converges to Au weakly in V', as $t \to 0$. A is called pseudo-monotone if A satisfies the following condition:

(2.2) If $\{u_n\}$ is a sequence such that u_n converges weakly to $u \in V$ and $\limsup\,(Au_n, u_n - u) \leq 0$, then

$$(Au, u - v) \leq \liminf_{n \to \infty} (Au_n, u_n - v) \quad \text{for each } v \in V.$$

Let E, F be Banach spaces, and let g be a mapping from E into F. We denote by E_w and F_w the spaces E and F endowed with their weak topologies, respectively. Then g is said to be weakly continuous if g is a continuous mapping from E_w into F_w. The mapping g is called demicontinuous if g is a continuous mapping from E into F_w. For each $r \geq 1$. We denote by $L^r(0,T;E)$ the space of E-valued measurable functions $u:(0,T) \to E$ such that $\int_0^T \|u(t)\|^r dt < \infty$. The pairing between $L^p(0,T;V)$ and $L^q(0,T;V')$ is denoted by $\langle\cdot,\cdot\rangle$. Then for each u, $v \in L^2(0,T;H)$, $\langle u, v\rangle$ is the ordinary innner product of u and v in $L^2(0,T;H)$. The norms of $L^p(0,T;V)$, $L^2(0,T;H)$, $L^q(0,T;V')$ are again denoted by $\|\cdot\|$, $|\cdot|$ and $\|\cdot\|_*$. We denote by J the duality mapping from $L^q(0,T;V')$ onto $L^p(0,T;V)$, i.e.,

(2.1) $J(u) = \{v \in L^p(0,T;V) : \langle v, u\rangle = \|v\|^2 = \|u\|_*^2\}$

for each $u \in L^q(0,T:V')$. By using the Asplund's renorming theorem, we may assume that J is a single valued monotone and demi-continuous mapping(cf. Proposition 2.14 of [3]). We will denote by L the operator defined by

$$(Lf)(t) = \int_0^t f(s)\,ds \qquad \text{for each } f \in L^2([0,T])$$

The adjoint operator L^* of L is given by

$$(L^*f)(t) = \int_t^T f(s)\,ds \qquad \text{for each } f \in L^2([0,T]).$$

Then L and L^* are positive operators on $L^2([0,T])$.

In the following we will assume that the mapping $A:V \to V'$ satisfies the following conditions:

(A1) A is a monotone hemicontinuous mapping from V into V';

(A2) there exist positive constants C_1, C_2 and C_3 such that

(2.3) $\|Ax\|_* \le C_1(\|x\|^{p-1} + 1)$, for each $x \in V$

and

(2.4) $C_2\|x\|^p \le C_3 + (Ax, x)$ for each $x \in V$.

We impose the following conditions on G:

(G1) G is a completely continuous mapping from V to V':

(G2) There exist positive constants a, b and C such that

$$(2.5) \qquad (G(x),\ x) \geq -C \qquad \text{for all } x \in V;$$

$$(2.6) \qquad \|G(x)\|_* \leq a\|x\|^{p-1} + b \qquad \text{for all } x \in V.$$

We now state our result:

Theorem . Suppose that (A1), (A2), (G1) and (G2) hold. Then for each $u_0 \in H$ and $f \in L^q(0,T;V')$, there exists a solution u of (1.1) such that

$$(2.7) \qquad u \in C(0,T;H) \cap L^p(0,T;V)$$

$$(2.8) \qquad \frac{du}{dt} \in L^q(0,T:V').$$

3. Propositions. Throughout this section, we assume that $u_0 \in V$, $f \in L^q(0,T;V')$, and that (A1), (A2), (G1) and (G2) hold We denote by $\widetilde{V}$, $\widetilde{H}$, and $\widetilde{V}'$ the spaces $L^p(0,T;V)$, $L^2(0,T;H)$ and $L^q(0,T;V')$, respectively. $\widetilde{A}$ denote the operator defined by

$$(\widetilde{A}u)(t) = A(u(t) + u_0) - f(t), \qquad \text{for each } u \in \widetilde{V} \text{ and } t \in (0,T].$$

Also we denote by $\widetilde{G}$ the mapping defined by

$$(\widetilde{G}u)(t) = G(u(t) + u_0) \qquad \text{for each } u \in \widetilde{V} \text{ and } t \in (0,T].$$

Then it is easy to see that $\widetilde{A}$ is a monotone hemicontinuous mapping satisfying the following conditions:

$$(3.1) \qquad \|\widetilde{A}u\|_* \leq c_1(1 + \|u\|^{p-1}) \qquad \text{for } u \in \widetilde{V};$$

$$(3.2) \qquad c_2\|u\|^p \leq c_3 + \langle\widetilde{A}u,\ u\rangle \qquad \text{for } u \in \widetilde{V},$$

where c_1, c_2 and c_3 are positive constants depending on C_1, C_2, C_3, T, u_0 and f. It is also easy to see that G is a continuous mapping from $\tilde{V}$ into $\tilde{H}$ satisfying that

(3.3) $\quad \langle \tilde{G}u, u\rangle \geq c \qquad$ for all $u \varepsilon \tilde{V}$;

(3.4) $\quad \|\tilde{G}u\|_* \leq \alpha\|u\|^{p-1} + \beta \qquad$ for all $u \varepsilon \tilde{V}$,

where c, α, β are constants depending on C, a, b and T and u_0.
We now consider the equation of the form

(3.5) $\quad v + (\tilde{A} + \tilde{G})Lv = 0$

Let $v \varepsilon \tilde{V}'$ be a solution of (3.5). Then it is easy to see that $u = Lv + u_0$ is a solution of (1.1). On the other hand, if u is a solution of (1.1), we have that $v = \frac{du}{dt}$ is a solution of (3.4). Since L^* is injective, the equation (3.5) is equivalent to

(3.6) $\quad L^*v + L^*(\tilde{A} + \tilde{G})Lv = 0.$

Then we will show the existence of the solutions of (3.6) instead of (1.1). In the rest of this section, we assume, for simplicity, that $u_0 = 0$ and $f = 0$. The proofs remains valid for each $u_0 \varepsilon V$ and $f \varepsilon \tilde{V}'$ with minor changes.

Proposition 1. the mapping $L^* + L^*(\tilde{A} + \tilde{G})L$ is a pseudo-monotone mapping from $\tilde{V}$ into $\tilde{V}'$.

Proof. From (A1), it is easily verified that $L^* + L^*\tilde{A}L:\tilde{V} \to \tilde{V}'$ is a monotone hemicontinuous mapping. Let $\{v_n\} \subset \tilde{V}$ be a sequence such that v_n converges to v weakly in $\tilde{V}$ and

(3.7) $\quad \limsup \langle L^*v_n + L^*(\tilde{A}+\tilde{G})Lv_n, v_n - v\rangle \leq 0.$

Since v_n converges to v weakly in $\tilde{V}$, we have that for each $t \in [0,T]$, $(Lv_n)(t)$ converges to $(Lv)(t)$ weakly in V. Since G is completely continuous, we find that $G((Lv_n)(t))$ converges to $G((Lv)(t))$ strongly in V' for all $t \in [0,T]$. Then noting that

$$\|G((Lv_n)(t))\|_* \leq a\|(Lv_n)(t)\|^{p-1} + b \leq a(T^{1/2}\sup \|v_n\|)^{p-1} + b$$

for each $t \in [0,T]$, we obtain by Lebesgue's bounded convergence theorem, that $\tilde{G}(Lv_n)$ converges to $\tilde{G}(Lv)$ strongly in $\tilde{V}'$. Thus we obtain that

(3.8) $\quad \langle\tilde{G}Lv, Lv\rangle = \lim \langle\tilde{G}Lv_n, Lv_n\rangle$

Therefore we have by (3.7) and (3.8) that

$$\limsup \langle L^*v_n + L^*\tilde{A}Lv_n, v_n - v\rangle \leq 0.$$

Then by lemma 1.3 of Chap II of [2], it follows that $L^*v_n + L^*\tilde{A}Lv_n$ converges to $L^*v + L^*\tilde{A}Lv$ weakly in $\tilde{V}'$ and

$$\langle L^*v + L^*\tilde{A}Lv, v\rangle = \lim \langle L^*v_n + L^*\tilde{A}Lv_n, v_n\rangle.$$

Then from (3.7), (3.8) and the equality above, we find that

$$\langle L^*v + L^*(\tilde{A}+\tilde{G})Lv, Lv - z\rangle \leq \liminf \langle L^*v_n + L^*(\tilde{A}+\tilde{G})Lv_n, v_n - z\rangle$$

for each $z \in \tilde{V}$. This completes the proof.

Proposition 2. Let $\{v_n\}$ be a sequence in $\tilde{V}'$ such that v_n converges to v weakly in $\tilde{V}'$, $\{Lv_n\} \subset \tilde{V}$, Lv_n converges to Lv weakly in $\tilde{V}$ and

$$\limsup \langle(\tilde{A}+\tilde{G})Lv_n, Lv_n - Lv\rangle \leq 0.$$

Then $(\tilde{A} + \tilde{G})Lv_n$ converges to $(\tilde{A} + \tilde{G})Lv$ weakly in $\tilde{V}'$, and

$$\lim \langle(\tilde{A}+\tilde{G})Lv_n, Lv_n\rangle = \langle(\tilde{A}+\tilde{G})Lv, Lv\rangle . \tag{3.9}$$

Proof. Let $\{v_n\}$ be a sequence in $\tilde{V}'$ satisfying the hypothesis of Proposition 2. Then by using (3.1) and (3.4), we can see that that $\{\|\tilde{A}Lv_n\|_*\}$ and $\{\|\tilde{G}Lv_n\|_*\}$ are bounded. We first show that

$$\liminf (A((Lv_n)(t)) + G((Lv_n)(t)), (Lv_n - Lv)(t)) \geq 0 \tag{3.10}$$

for all $t \in (0,T)$. Suppose that for some $t \in (0,T)$,

$$\liminf (A((Lv_n)(t)) + G((Lv_n)(t)), (Lv_n - Lv)(t)) < 0. \tag{3.11}$$

From (A2) and (G2), we have that

$$\begin{aligned}(A((Lv_n)(t)) &+ G((Lv_n)(t)), (Lv_n - Lv)(t)) \\ &\geq C_2\|(Lv_n)(t)\|^p - C_3 - C - C_1(1 + \|(Lv_n)(t)\|^{p-1})\|(Lv)(t)\| \\ &\qquad - (a\|(Lv_n)(t)\|^{p-1} + b)\|(Lv)(t)\|.\end{aligned} \tag{3.12}$$

Then it follows from (3.11) and (3.12) that $\{\|(Lv_n)(t)\|\}$ is bounded. Then since G is compltetly continuous, $G(Lv_n)(t)$ converges to $G(Lv)(t)$ strongly in V'. Therefore we have that $\lim (G((Lv_n)(t)), (Lv_n)(t) - (Lv)(t)) = 0$. On the other hand, we have from the monotonicity of A that

$$\liminf (A((Lv_n)(t)), (Lv_n - Lv)(t)) \geq 0.$$

for all $t \in (0,T)$. Then we have that

$$\liminf\, (A((Lv_n)(t)) + G((Lv_n)(t)),\ (Lv_n - Lv)(t)) \geq 0.$$

This contradicts to (3.11). Thus we have shown that (3.10) holds for all $t \in (0,T)$. We can see from (3.12) that

$$h_n(t) = (A((Lv_n)(t)) + G((Lv_n)(t)),\ (Lv_n - Lv)(t))$$
$$\geq K_1\|Lv(t)\|^p + K_2$$

for all $t \in (0,T)$ and $n \geq 1$, where K_1, K_2 are constants depending on C, C_1, C_2, C_3, a, and b. Then by Fatou's lemma, we have that

$$(3.13)\quad 0 = \int_0^T \liminf h_n(t)\,dt$$
$$\leq \liminf \int_0^T h_n(t)\,dt \leq \limsup \langle(\tilde{A}+\tilde{G})Lv_n,\ Lv_n - Lv\rangle \leq 0.$$

The inequality above implies that $\lim \int_0^T |h_n|\,dt = 0$. Then we can choose a subsequence $\{h_{n_i}\}$ of $\{h_n\}$ such that

$$(3.14)\quad \lim\, (A((Lv_{n_i})(t)) + G((Lv_{n_i})(t)),\ (Lv_{n_i} - Lv)(t)) = 0,$$

a.e. $t \in (0,T)$. By (3.12) and (3.14), we find that $\{\|Lv_{n_i}(t)\|\}$ is bounded for a.e. $t \in (0,T)$. Since $(Lv_{n_i})(t)$ converges to $(Lv)(t)$ weakly in V', we have that $G(Lv_{n_i})(t)$ converges to $G(Lv)(t)$ strongly in H, for a.e. $t \in (0,T)$. Therefore it follows from (2.6) that

$\lim\, (G((Lv_{n_i})(t)),\ (Lv_{n_i})(t) - (Lv)(t)) = 0$. Then we have

$$\lim\, (A((Lv_{n_i})(t)),\ (Lv_{n_i} - Lv)(t)) = 0 \quad \text{a.e. } t \in (0,T).$$

Then since A is monotone, we have from lemma 1.3 of Chap. II of [2] that $A(Lu_{n_i}(t))$ converges to $A((Lv)(t))$ weakly in V'. Here we observe by using (3.1)-(3.4) that for each $z \in \tilde{V}$ and $t \in (0,T)$, there exist real numbers K_3, K_4 such that

(3.15) $\quad ((A + G)((Lv_n)(t)), (Lv_n)(t) - z(t)) \geq K_3\|z(t)\|^p + K_4$

for each $n \geq 1$. Then from Fatou's lemma, we find that

$$
\begin{aligned}
(3.16)\quad & <(\tilde{A} + \tilde{G})Lv,\ Lv - z> \\
& = \int_0^T \lim\ (A((Lv_{n_i})(t)) + G((Lv_{n_i})(t)),\ (Lv_{n_i})(t) - z(t))\ dt \\
& \leq \liminf <(\tilde{A} + \tilde{G})Lv_{n_i},\ Lv_{n_i} - z> \\
& \leq \limsup <(\tilde{A} + \tilde{G})Lv_n,\ (Lv_n - Lv) + (Lv - z)> \\
& \leq \limsup <(\tilde{A} + \tilde{G})Lv_n,\ Lv - z>, \qquad \text{for all } z \in \tilde{V}.
\end{aligned}
$$

The inequality above implies that $(\tilde{A}+\tilde{G})Lv_n$ converges to $(\tilde{A}+\tilde{G})Lv$ weakly in $\tilde{V}'$. We also obtain from (3.13) that (3.9) holds.

Proposition 3. For each $k > 0$, the equation

(3.17) $\quad L^*v + kJv + L^*(\tilde{A} + \tilde{G})Lv = 0$

has a solution $v \in \tilde{V}'$.

Proof. Let $B_r = \{v \in \tilde{V}: \|v\| \leq r\}$ for $r > 0$. Since the mapping $L^* + L^*(\tilde{A}+\tilde{G})L : \tilde{V} \to \tilde{V}'$ is pseudo-monotone and $J: \tilde{V} \to \tilde{V} (\subset \tilde{V}')$ is monotone hemicontinuous, we can see that the sum $kJ + L^* + L^*(\tilde{A}+\tilde{G})L$ is also pseudo-monotone (cf. Proposition 23 of (4)), for each $k > 0$. Then we have ,by using theorem 7.8 of (6), that for each $n \geq 1$, there exists a solution $v_n \in B_n$ of the inequality

$$(3.18) \qquad \langle L^* v_n + kJv_n + L^*(\tilde{A}+\tilde{G})Lv_n,\ z - v_n\rangle \geq 0 \qquad \text{for all } z \in B_n.$$

The inequlity (3.18) implies that for each $m \geq 1$,

$$(3.19) \qquad \limsup_{n\to\infty} (\langle v_n + (\tilde{A}+\tilde{G})Lv_n,\ Lv_n - Lv_m\rangle + k\langle Jv_n,\ v_n - v_m\rangle)$$

$$= \limsup_{n\to\infty} \langle L^* v_n + kJv_n + L^*(\tilde{A}+\tilde{G})Lv_n,\ v_n - v_m\rangle \leq 0.$$

By putting $v = 0$ in (3.18), we have

$$(3.20) \qquad k\|v_n\|_*^2 + c_2\|Lv_n\|^p + c \leq c_3 \qquad \text{for all } n \geq 1.$$

The inequality above implies that $\{\|v_n\|_*\}$ and $\{\|Lv_n\|\}$ are bounded. Then we may assume without any loss of generality that v_n converges to $v \in \tilde{V}'$ weakly in $\tilde{V}'$ and Lv_n converges to Lv weakly in $\tilde{V}$. Then from (3.19), it is easy to see that

$$(3.21) \qquad \limsup (\langle v_n + (\tilde{A}+\tilde{G})Lv_n,\ Lv_n - Lv\rangle + k\langle Jv_n,\ v_n - v\rangle) \leq 0.$$

Here we choose a sequence $\{z_n\} \subset \tilde{V}'$ such that $z_n \in co\{v_n\}$, z_n converges to v strongly in $\tilde{V}'$ and Lz_n converges to Lv strongly in $\tilde{V}$. Then since $\langle Lv_n - Lz_m,\ v_n - z_m\rangle \geq 0$, we find, by letting $m, n \to \infty$ that

$$(3.22) \qquad \liminf \langle v_n,\ Lv_n - Lv\rangle \geq 0.$$

Also we have by the monotonicity of J that

(3.23) $\liminf \langle Jv_n, v_n - v\rangle \geq 0.$

Combining (3.22) and (3.23) with (3.21), we have

(3.23) $\limsup \langle(\tilde{A} + \tilde{G})Lv_n, Lv_n - Lv\rangle \leq 0.$

Then we obtain by Proposition 2 that $(\tilde{A}+\tilde{G})Lv_n$ converges to $(\tilde{A}+\tilde{G})Lv$ weakly in $\tilde{V}'$ and

(3.24) $\lim \langle(\tilde{A} + \tilde{G})Lv_n, Lv_n\rangle = \langle(\tilde{A} + \tilde{G})Lv, Lv\rangle.$

Then the inequality (3.18) implies that

(3.25) $\langle L^*v + kJv + L^*(\tilde{A}+\tilde{G})Lv, z\rangle \geq \langle(\tilde{A}+\tilde{G})Lv, Lv\rangle$ for all $z \in \tilde{V}$.

Since $z \in \tilde{V}$ is arbitrary, we find that $L^*v + kJv + L^*(\tilde{A}+\tilde{G})Lv = 0.$

4. Proof of Theorems. In the following, we assume that (A1), (A2), (G1) and (G2) are satisfied. We first show that the assertion of Theorem 1 holds for each $u_0 \in V$ and $f \in \tilde{V}'$.

Let $u_0 \in V$, $f \in \tilde{V}'$ and let $\tilde{A}$, $\tilde{G}$ be as in section 3. Then by Proposition 3, there exists a solution $v_n \in \tilde{V}'$ of the equation

(4.1) $L^*v_n + \frac{1}{n}Jv_n + L^*(\tilde{A}+\tilde{G})Lv_n = 0$

for each $n \geq 1$. Multiplying (4.1) by v_n, we find that

(4.2) $\frac{1}{n}\|v_n\|_*^2 + c_2\|Lv_n\|^p + c \leq c_3$ for each $n \geq 1$.

From (4.2), we have that $\{\|Lv_n\|\}$ is bounded. Then it follows from (3.1) and (3.4) that $\{\|\tilde{A}Lv_n\|_*\}$ and $\{\|\tilde{G}Lv_n\|_*\}$ are bounded. It also follows from (4.2) that $\lim \|\frac{1}{n}Jv_n\| = 0$. Since L^* is injective in $\tilde{V}'$, the equation (4.1) can be rewritten as

(4.3) $$v_n + \frac{1}{n}(L^*)^{-1}Jv_n + (\tilde{A}+\tilde{G})Lv_n = 0 \quad \text{for each } n \geq 1.$$

Here we note that $<(L^*)^{-1}Jv_n, Jv_n> \geq 0$ for $n \geq 1$. Then multiplying (4.3) by Jv_n, we have

(4.4) $$\|v_n\|_*^2 \leq \|(\tilde{A}+\tilde{G})Lv_n\|_*\|Jv_n\| \leq (\|\tilde{A}Lv_n\|_* + \|\tilde{G}Lv_n\|_*)\|v_n\|_*.$$

for $n \geq 1$. Thus we find that $\{\|v_n\|_*\}$ is bounded. Then we may suppose without any loss of generality that v_n converges to $v \in \tilde{V}'$ weakly in $\tilde{V}'$ and Lv_n converges to Lv weakly in $\tilde{V}$.

While, we have by multiplying (4.1) by $v_n - v_m$ that

(4.5) $$<v_n + (\tilde{A}+\tilde{G})Lv_n, Lv_n - Lv_m> + <\frac{1}{n}Jv_n, v_n - v_m> = 0.$$

Then since $\frac{1}{n}Jv_n$ converges to 0 in $\tilde{V}$ as $n \to \infty$, we find that

(4.6) $$\lim_{n\to\infty} <v_n + (\tilde{A}+\tilde{G})Lv_n, Lv_n - Lv_m> = 0 \quad \text{for each } m \geq 1.$$

Then it is easy to see from (4.6) that

(4.7) $$\lim <v_n + (\tilde{A}+\tilde{G})Lv_n, Lv_n - Lv> = 0.$$

Since $\liminf (v_n, Lv_n - Lv> \geq 0$, we have by (4.7) that

(4.8) $$\limsup <(\tilde{A}+\tilde{G})Lv_n, Lv_n - Lv> \leq 0.$$

Then by Propostion 2, we find that $(\tilde{A}+\tilde{G})Lv_n$ converges to $(\tilde{A}+\tilde{G})Lv$ weakly in $\tilde{V}'$. Therefore we obtain from (4.1) that $L^*v + L^*(\tilde{A}+\tilde{G})Lv = 0$, i.e., (1.1) has a solution.

Now let $u_0 \in H$ and $\{u_n^0\} \subset V$ be a sequece such that u_n^0 converges to u_0 strongly in H. Then by the argument above, we have that for each $n \geq 1$, there exists a solution u_n of the problem

$$(4.9)\quad \begin{aligned} &\frac{du_n}{dt} + Au_n + G(u_n) = f, \quad 0 < t < T, \\ &u(0) = u_n^0. \end{aligned}$$

Here we assume for simplicity that $f = 0$. Then multiplying (4.9) by u_n and integrating, we have

$$(4.10)\quad \frac{1}{2}|u_n(t)|^2 + C_2\int_0^T \|u_n(s)\|^p ds < (C + C_3)T + \sup_n |u_n^0|^2.$$

Then $\{u_n\}$ is bounded in $\tilde{V}$. Also by (2.3) and (2.6), we see that $\{\frac{du_n}{dt}\}$ is bounded in $\tilde{V}'$. Here we put $v_n = \frac{du_n}{dt}$ for each $n \geq 1$. Then from the observation above, we may suppose that v_n converges to $v \in \tilde{V}'$ weakly in $\tilde{V}'$ and $u_n = Lv_n + u_n^0$ converges to $u = Lv + u_0$ weakly in $\tilde{V}$. We set $(\bar{A}z)(t) = A(z(t))$ and $(\bar{G}z)(t) = G(z(t))$ for each $z \in \tilde{V}$ and $t \in (0,T)$. Now we multiply (4.9) by $u_n - u$ and integrate. Then we have

$$(4.11)\quad \limsup \langle(\bar{A} + \bar{G})(Lv_n + u_n^0),\ (Lv_n + u_n^0) - (Lv + u_0)\rangle$$

$$= \limsup \langle\frac{du_n}{dt},\ u - u_n\rangle$$

$$\leq \limsup (-|u(T) - u_n(T)|^2 + |u_0 - u_n^0|^2 + \langle\frac{du}{dt},\ u_n - u\rangle) \leq 0.$$

Therefore the hypothesis of Proposition 2 is satisfied with Lv_n replaced by $Lv_n + u_n^0$ and Lv replaced by $Lv + u_0$. It is easy to verify that the proof of Propostition 2 remains valid for A, G, Lv and Lv_n replaced by $\overline{A}$, $\overline{G}$, $Lv + u_0$ and $Lv_n + u_n^0$, respectively. Therefore we find that $(\overline{A}+\overline{G})(Lv_n + u_n^0)$ converges to $(\overline{A}+\overline{G})(Lv + u_0)$ weakly in $\tilde{V}$. Thus we obtain $v + (\overline{A}+\overline{G})(Lv + u_0) = 0$. This implies that $u = Lv + u_0$ is a solution of (1.1). We can see that $u \in C(0,T;H)$ by the usual argument(cf. theorem 4.5 of [3]),

REFERENCES

[1] Attouch A. & Damlamian A., On multivalued evolution equations in Hilbert spaces, Israel J. Math., 12(1972), 373-390.

[2] Barbu V., Nonlinear semigroups and evolution equations in Banach spaces, Noordhoff, Leyden (1976).

[3] Barbu V., & Precupanu Th., Convexity and Optimization in Banach Spaces, Editura Academiei R.S.R., Bucharest(1975).

[4] Brezis H., Equations et inequations non lineaires dans les espaces vectoriels en dualite, Ann. Inst. Fourier. Grenoble 18(1968), 115-175.

[5] Browder F. E., Non-linear equations of evolution, Ann. Math., 80(1964), 485-523.

[6] Browder F. E., Nonlinear Operators and Nonlinear Equations of Evolution in Banach spaces, Proceedings of Symposia in Pure Math.,XVIII, Part 2(1976).

[7] Crandall M. G. & Nohel J. A., An abstract functional differential equation and a related Volterra equation, Israel

J. Math., 29(1979), 313-328.

[8] Gutman, S., Compact perturbations of m-accretive operators in general Banach spaces, SIAM J. Math., 13(1982), 789-800.

[9] Gutman, S., Evolutions governed by m-accretive plus compact operators, Nonlinear Analysis TMA, 7(1983), 707-715.

[10] Hirano, N., Local existence theorems for nonlinear differential equations, SIAM J. Math. Anal., 14(1983), 117-125.

[11] Hirano, N., Abstract nonlinear Volterra equations with positive kernels, SIAM J. Math. Anal., 17(1986), 403-414.

[12] Pavel N. H., Invariant sets for a class of semi linear equations of evolutions, Nonlinear Analysis TMA, 1(1977), 187-196.

[13] Pavel N. H., Equations d'evolution multivoques dans des espaces de Banach, C. r. hebd. Seanc. Acad. Sci. Paris 287(1978), 315-317.

[14] Pazy, A., A class of semi-linear equations of evolution, Israel J. Math., 20(1975), 23-36.

[15] Vrabie, I. I., The nonlinear version of Pazy's local existence theorem, Israel J. Math., 32(1979), 221-235.

[16] Vrabie, I. I., An existence result for a class of nonlinear evolution equations in Banach spaces, Nonlinear Analysis, 6(1982), 711-722.

[17] Vrabie, I. I., Compactness methods for an abstract nonlinear Volterra integrodifferential equations, Nonlinear Analysis TMA, 5(1981), 355-371.